Postulates, Theorems, and Constructions

Chapter 1 Tools of Geometry

Postulate 1-1
Through any two points there is exactly one line. (p. 13)

Postulate 1-2
If two distinct lines intersect, then they intersect in exactly one point. (p. 13)

Postulate 1-3
If two distinct planes intersect, then they intersect in exactly one line. (p. 14)

Postulate 1-4
Through any three noncollinear points there is exactly one plane. (p. 15)

Postulate 1-5
Ruler Postulate
Every point on a line can be paired with a real number. This makes a one-to-one correspondence between the points on the line and the real numbers. (p. 20)

Postulate 1-6
Segment Addition Postulate
If three points A, B, and C are collinear and B is between A and C, then $AB + BC = AC$. (p. 21)

Postulate 1-7
Protractor Postulate
Consider $\overrightarrow{OB}$ and a point A on one side of $\overrightarrow{OB}$. Every ray of the form $\overrightarrow{OA}$ can be paired one to one with a real number from 0 to 180. (p. 28)

Postulate 1-8
Angle Addition Postulate
If point B is in the interior of $\angle AOC$, then $m\angle AOB + m\angle BOC = m\angle AOC$. (p. 30)

Postulate 1-9
Linear Pair Postulate
If two angles form a linear pair, then they are supplementary. (p. 36)

The Midpoint Formulas
On a Number Line
The coordinate of the midpoint M of $\overline{AB}$ is $\frac{a+b}{2}$.
In the Coordinate Plane
Given $\overline{AB}$ where $A(x_1, y_1)$ and $B(x_2, y_2)$, the coordinates of the midpoint of $\overline{AB}$ are $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. (p. 50)

The Distance Formula
The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. (p. 52)
Proof on p. 497, Exercise 35

The Distance Formula (Three Dimensions)
In a three-dimensional coordinate system, the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) can be found with this extension of the Distance Formula.
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ (p. 56)

Postulate 1-10
Area Addition Postulate
The area of a region is the sum of the areas of its nonoverlapping parts. (p. 63)

Chapter 2 Reasoning and Proof

Law of Detachment
If the hypothesis of a true conditional is true, then the conclusion is true. In symbolic form:
If $p \to q$ is true and p is true, then q is true. (p. 106)

Law of Syllogism
If $p \to q$ is true and $q \to r$ is true, then $p \to r$ is true. (p. 108)

Properties of Congruence
Reflexive Property
$\overline{AB} \cong \overline{AB}$ and $\angle A \cong \angle A$

Symmetric Property
If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

Transitive Property
If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.
If $\angle A \cong \angle B$, and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.
If $\angle B \cong \angle A$, and $\angle B \cong \angle C$, then $\angle A \cong \angle C$. (p. 114)

Theorem 2-1
Vertical Angles Theorem
Vertical angles are congruent. (p. 120)
Proof on p. 121

Theorem 2-2
Congruent Supplements Theorem
If two angles are supplements of the same angle (or of congruent angles), then the two angles are congruent. (p. 122)
Proof on p. 123, Problem 3

Theorem 2-3

Congruent Complements Theorem

If two angles are complements of the same angle (or of congruent angles), then the two angles are congruent. (p. 123)

- Proof on p. 125, Exercise 13

Theorem 2-4

All right angles are congruent. (p. 123)

- Proof on p. 125, Exercise 18

Theorem 2-5

If two angles are congruent and supplementary, then each is a right angle. (p. 123)

- Proof on p. 126, Exercise 23

Chapter 3 Parallel and Perpendicular Lines

Theorem 3-2

Corresponding Angles Theorem

If a transversal intersects two parallel lines, then corresponding angles are congruent. (p. 149)

- Proof on p. 155, Exercise 25

Theorem 3-1

Alternate Interior Angles Theorem

If a transversal intersects two parallel lines, then alternate interior angles are congruent. (p. 149)

- Proof on p. 150

Postulate 3-1

Same-Side Interior Angles Postulate

If a transversal intersects two parallel lines, then same-side interior angles are supplementary. (p. 148)

Theorem 3-3

Alternate Exterior Angles Theorem

If a transversal intersects two parallel lines, then alternate exterior angles are congruent. (p. 151)

- Proof on p. 150, Got It 2

Theorem 3-4

Converse of the Corresponding Angles Theorem

If two lines and a transversal form corresponding angles that are congruent, then the lines are parallel. (p. 156)

- Proof on p. 161, Exercise 29

Theorem 3-5

Converse of the Alternate Interior Angles Theorem

If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel. (p. 157)

- Proof on p. 158

Theorem 3-6

Converse of the Same-Side Interior Angles Postulate

If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel. (p. 157)

- Proof on p. 158, Got It 2

Theorem 3-7

Converse of the Alternate Exterior Angles Theorem

If two lines and a transversal form alternate exterior angles that are congruent, then the two lines are parallel. (p. 157)

- Proof on p. 158, Problem 2

Theorem 3-8

If two lines are parallel to the same line, then they are parallel to each other. (p. 164)

- Proof on p. 167, Exercise 7

Theorem 3-9

In a plane, if two lines are perpendicular to the same line, then they are parallel to each other. (p. 165)

- Proof on p. 165

Theorem 3-10

Perpendicular Transversal Theorem

In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other. (p. 166)

- Proof on p. 168, Exercise 10

Postulate 3-2

Parallel Postulate

Through a point not on a line, there is one and only one line parallel to the given line. (p. 171)

Theorem 3-11

Triangle Angle-Sum Theorem

The sum of the measures of the angles of a triangle is 180. (p. 172)

- Proof on p. 172

Theorem 3-12

Triangle Exterior Angle Theorem

The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles. (p. 173)

- Proof on p. 177, Exercise 33

Corollary

The measure of an exterior angle of a triangle is greater than the measure of each of its remote interior angles. (p. 325)

- Proof on p. 325

Spherical Geometry Parallel Postulate

Through a point not on a line, there is no line parallel to the given line. (p. 179)

Postulate 3-3

Perpendicular Postulate

Through a point not on a line, there is one and only one line perpendicular to the given line. (p. 184)

Reference

Slopes of Parallel Lines

If two nonvertical lines are parallel, then their slopes are equal. If the slopes of two distinct nonvertical lines are equal, then the lines are parallel. Any two vertical lines or horizontal lines are parallel. (p. 197)

- Proofs on p. 457, Exercises 33, 34

Slopes of Perpendicular Lines

If two nonvertical lines are perpendicular, then the product of their slopes is -1. If the slopes of two lines have a product of -1, then the lines are perpendicular. Any horizontal line and vertical line are perpendicular. (p. 198)

- Proofs on p. 418, Exercise 28; p. 497, Exercise 51; p. 466, Exercise 44

Chapter 4 Congruent Triangles

Theorem 4-1

Third Angles Theorem

If the two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent. (p. 220)

- Proof on p. 220

Postulate 4-1

Side-Side-Side (SSS) Postulate

If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent. (p. 227)

Postulate 4-2

Side-Angle-Side (SAS) Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent. (p. 228)

Postulate 4-3

Angle-Side-Angle (ASA) Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent. (p. 234)

Theorem 4-2

Angle-Angle-Side (AAS) Theorem

If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the triangles are congruent. (p. 236)

- Proof on p. 236

Theorem 4-3

Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles opposite those sides are congruent. (p. 250)

- Proofs on p. 251; p. 255, Exercise 22

Corollary

If a triangle is equilateral, then the triangle is equiangular. (p. 252)

- Proof on p. 255, Exercise 24

Theorem 4-4

Converse of the Isosceles Triangle Theorem

If two angles of a triangle are congruent, then the sides opposite the angles are congruent. (p. 251)

- Proof on p. 255, Exercise 23

Corollary

If a triangle is equiangular, then the triangle is equilateral. (p. 252)

- Proof on p. 255, Exercise 24

Theorem 4-5

If a line bisects the vertex angle of an isosceles triangle, then the line is also the perpendicular bisector of the base. (p. 252)

- Proof on p. 255, Exercise 26

Theorem 4-6

Hypotenuse-Leg (HL) Theorem

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent. (p. 259)

- Proof on p. 259

Chapter 5 Relationships Within Triangles

Theorem 5-1

Triangle Midsegment Theorem

If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side and is half as long. (p. 285)

- Proof on p. 415, Got It 2

Theorem 5-2

Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment. (p. 293)

- Proof on p. 298, Exercise 32

Theorem 5-3

Converse of the Perpendicular Bisector Theorem

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment. (p. 293)

- Proof on p. 298, Exercise 33

Theorem 5-4

Angle Bisector Theorem

If a point is on the bisector of an angle, then the point is equidistant from the sides of the angle. (p. 295)

- Proof on p. 298, Exercise 34

Theorem 5-5

Converse of the Angle Bisector Theorem

If a point in the interior of an angle is equidistant from the sides of the angle, then the point is on the angle bisector. (p. 295)

- Proof on p. 298, Exercise 35

Theorem 5-6

Concurrency of Perpendicular Bisectors Theorem
The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices. (p. 301)
- Proof on p. 302

Theorem 5-7

Concurrency of Angle Bisectors Theorem
The bisectors of the angles of a triangle are concurrent at a point equidistant from the sides of the triangle. (p. 303)
- Proof on p. 306, Exercise 24

Theorem 5-8

Concurrency of Medians Theorem
The medians of a triangle are concurrent at a point that is two-thirds the distance from each vertex to the midpoint of the opposite side. (p. 309)
- Proof on p. 417, Exercise 25

Theorem 5-9

Concurrency of Altitudes Theorem
The lines that contain the altitudes of a triangle are concurrent. (p. 310)
- Proof on p. 417, Exercise 26

Comparison Property of Inequality

If $a = b + c$ and $c > 0$, then $a > b$. (p. 324)
- Proof on p. 324

Theorem 5-10

If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side. (p. 325)
- Proof on p. 330, Exercise 40

Theorem 5-11

If two angles of a triangle are not congruent, then the longer side lies opposite the larger angle. (p. 326)
- Proof on p. 326

Theorem 5-12

Triangle Inequality Theorem
The sum of the lengths of any two sides of a triangle is greater than the length of the third side. (p. 327)
- Proof on p. 331, Exercise 45

Theorem 5-13

The Hinge Theorem (SAS Inequality Theorem)
If two sides of one triangle are congruent to two sides of another triangle and the included angles are not congruent, then the longer third side is opposite the larger included angle. (p. 332)
- Proof on p. 338, Exercise 25

Theorem 5-14

Converse of the Hinge Theorem (SSS Inequality)
If two sides of one triangle are congruent to two sides of another triangle and the third sides are not congruent, then the larger included angle is opposite the longer third side. (p. 334)
- Proof on p. 334

Chapter 6 Polygons and Quadrilaterals

Theorem 6-1

Polygon Angle-Sum Theorem
The sum of the measures of the angles of an n-gon is $(n - 2)180$. (p. 353)
- Proof on p. 357, Exercise 40

Corollary
The measure of each angle of a regular n-gon is $\frac{(n-2)180}{n}$. (p. 354)
- Proof on p. 358, Exercise 43

Theorem 6-2

Polygon Exterior Angle-Sum Theorem
The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360. (p. 355)
- Proofs on p. 352 (using a computer); p. 357, Exercise 39

Theorem 6-3

If a quadrilateral is a parallelogram, then its opposite sides are congruent. (p. 359)
- Proof on p. 360

Theorem 6-4

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary. (p. 360)
- Proof on p. 365, Exercise 32

Theorem 6-5

If a quadrilateral is a parallelogram, then its opposite angles are congruent. (p. 361)
- Proof on p. 361, Problem 2

Theorem 6-6

If a quadrilateral is a parallelogram, then its diagonals bisect each other. (p. 362)
- Proof on p. 364, Exercise 13

Theorem 6-7

If three (or more) parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal. (p. 363)
- Proof on p. 366, Exercise 43

Theorem 6-8

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (p. 367)
- Proof on p. 373, Exercise 20

Theorem 6-9

If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram. (p. 368)
- Proof on p. 373, Exercise 21

Theorem 6-10

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (p. 368)
- Proof on p. 373, Exercise 18

Theorem 6-11
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. (p. 369)
Proof on p. 369

Theorem 6-12
If one pair of opposite sides of a quadrilateral is both congruent and parallel, then the quadrilateral is a parallelogram. (p. 370)
Proof on p. 373, Exercise 19

Theorem 6-13
If a parallelogram is a rhombus, then its diagonals are perpendicular. (p. 376)
Proof on p. 377

Theorem 6-14
If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles. (p. 376)
Proof on p. 381, Exercise 45

Theorem 6-15
If a parallelogram is a rectangle, then its diagonals are congruent. (p. 378)
Proof on p. 381, Exercise 41

Theorem 6-16
If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (p. 383)
Proof on p. 383

Theorem 6-17
If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus. (p. 384)
Proof on p. 387, Exercise 23

Theorem 6-18
If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle. (p. 384)
Proof on p. 387, Exercise 24

Theorem 6-19
If a quadrilateral is an isosceles trapezoid, then each pair of base angles is congruent. (p. 389)
Proof on p. 396, Exercise 45

Theorem 6-20
If a quadrilateral is an isosceles trapezoid, then its diagonals are congruent. (p. 391)
Proof on p. 396, Exercise 53

Theorem 6-21
Trapezoid Midsegment Theorem
If a quadrilateral is a trapezoid, then
(1) the midsegment is parallel to the bases, and
(2) the length of the midsegment is half the sum of the lengths of the bases. (p. 391)
Proofs on p. 409, Problem 3; p. 415, Problem 2

Theorem 6-22
If a quadrilateral is a kite, then its diagonals are perpendicular. (p. 392)
Proof on p. 392

Chapter 7 Similarity

Postulate 7-1
Angle-Angle Similarity (AA ~) Postulate
If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar. (p. 450)

Theorem 7-1
Side-Angle-Side Similarity (SAS ~) Theorem
If an angle of one triangle is congruent to an angle of a second triangle, and the sides that include the two angles are proportional, then the triangles are similar. (p. 451)
Proof on p. 457, Exercise 35

Theorem 7-2
Side-Side-Side Similarity (SSS ~) Theorem
If the corresponding sides of two triangles are proportional, then the triangles are similar. (p. 451)
Proof on p. 458, Exercise 36

Theorem 7-3
The altitude to the hypotenuse of a right triangle divides the triangle into two triangles that are similar to the original triangle and to each other. (p. 460)
Proof on p. 461

Corollary 1
The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse. (p. 462)
Proof on p. 466, Exercise 42

Corollary 2
The altitude to the hypotenuse of a right triangle separates the hypotenuse so that the length of each leg of the triangle is the geometric mean of the length of the hypotenuse and the length of the segment of the hypotenuse adjacent to the leg. (p. 463)
Proof on p. 466, Exercise 43

Theorem 7-4
Side-Splitter Theorem
If a line is parallel to one side of a triangle and intersects the other two sides, then it divides those sides proportionally. (p. 471)
Proof on p. 472

Converse
If a line divides two sides of a triangle proportionally, then it is parallel to the third side.
Proof on p. 476, Exercise 37

Corollary
If three parallel lines intersect two transversals, then the segments intercepted on the transversals are proportional. (p. 473)
Proof on p. 477, Exercise 46

Theorem 7-5

Triangle-Angle-Bisector Theorem

If a ray bisects an angle of a triangle, then it divides the opposite side into two segments that are proportional to the other two sides of the triangle. (p. 473)

Proof on p. 477, Exercise 47

Chapter 8 Right Triangles and Trigonometry

Theorem 8-1

Pythagorean Theorem

If a triangle is a right triangle, then the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$a^2 + b^2 = c^2$ (p. 491)

Proof on p. 497, Exercise 49

Theorem 8-2

Converse of the Pythagorean Theorem

If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle. (p. 493)

Proof on p. 498, Exercise 52

Theorem 8-3

If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is obtuse. (p. 494)

Proof on p. 498, Exercise 53

Theorem 8-4

If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is acute. (p. 494)

Proof on p. 498, Exercise 54

Theorem 8-5

45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, both legs are congruent and the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

hypotenuse $= \sqrt{2} \cdot$ leg (p. 499)

Proof on p. 499

Theorem 8-6

30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.

hypotenuse $= 2 \cdot$ shorter leg

longer leg $= \sqrt{3} \cdot$ shorter leg (p. 501)

Proof on p. 501

Law of Sines

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ (p. 522)

Proof on p. 522

Law of Cosines

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$ (p. 527)

Proof on p. 527

Chapter 9 Transformations

Theorem 9-1

The composition of two or more isometries is an isometry. (p. 570)

Theorem 9-2

Reflections Across Parallel Lines

A composition of reflections across two parallel lines is a translation. (p. 571)

Theorem 9-3

Reflections Across Intersecting Lines

A composition of reflections across two intersecting lines is a rotation. (p. 572)

Chapter 10 Area

Theorem 10-1

Area of a Rectangle

The area of a rectangle is the product of its base and height.

$A = bh$ (p. 616)

Theorem 10-2

Area of a Parallelogram

The area of a parallelogram is the product of a base and the corresponding height.

$A = bh$ (p. 616)

Theorem 10-3

Area of a Triangle

The area of a triangle is half the product of a base and the corresponding height.

$A = \frac{1}{2}bh$ (p. 618)

Theorem 10-4

Area of a Trapezoid

The area of a trapezoid is half the product of the height and the sum of the bases.

$A = \frac{1}{2}h(b_1 + b_2)$ (p. 623)

Theorem 10-5

Area of a Rhombus or a Kite

The area of a rhombus or a kite is half the product of the lengths of its diagonals.

$A = \frac{1}{2}d_1d_2$ (p. 624)

Reference

Postulate 10-1

If two figures are congruent, then their areas are equal. (p. 630)

Theorem 10-6

Area of a Regular Polygon

The area of a regular polygon is half the product of the apothem and the perimeter.

$A = \frac{1}{2}ap$ (p. 630)

Proof on p. 630

Theorem 10-7

Perimeters and Areas of Similar Figures

If the scale factor of two similar figures is $\frac{a}{b}$, then

(1) the ratio of their perimeters is $\frac{a}{b}$ and

(2) the ratio of their areas is $\frac{a^2}{b^2}$. (p. 635)

Theorem 10-8

Area of a Triangle Given SAS

The area of a triangle is half the product of the lengths of two sides and the sine of the included angle.

Area of $\triangle ABC = \frac{1}{2}bc(\sin A)$ (p. 645)

Proof on p. 645

Postulate 10-2

Arc Addition Postulate

The measure of the arc formed by two adjacent arcs is the sum of the measures of the two arcs.

$m\widehat{ABC} = m\widehat{AB} + m\widehat{BC}$ (p. 650)

Theorem 10-9

Circumference of a Circle

The circumference of a circle is π times the diameter.

$C = \pi d$ or $C = 2\pi r$ (p. 651)

Theorem 10-10

Arc Length

The length of an arc of a circle is the product of the ratio $\frac{\text{measure of the arc}}{360}$ and the circumference of the circle.

length of $\widehat{AB} = \frac{m\widehat{AB}}{360} \cdot 2\pi r$ or

length of $\widehat{AB} = \frac{m\widehat{AB}}{360} \cdot \pi d$ (p. 653)

Theorem 10-11

Area of a Circle

The area of a circle is the product of π and the square of the radius.

$A = \pi r^2$ (p. 660)

Theorem 10-12

Area of a Sector of a Circle

The area of a sector of a circle is the product of the ratio $\frac{\text{measure of the arc}}{360}$ and the area of the circle.

Area of sector $AOB = \frac{m\widehat{AB}}{360} \cdot \pi r^2$ (p. 661)

Chapter 11 Surface Area and Volume

Theorem 11-1

Lateral and Surface Areas of a Prism

The lateral area of a right prism is the product of the perimeter of the base and the height of the prism.

L.A. $= ph$

The surface area of a right prism is the sum of the lateral area and the areas of the two bases.

S.A. $=$ L.A. $+ 2B$ (p. 700)

Theorem 11-2

Lateral and Surface Areas of a Cylinder

The lateral area of a right cylinder is the product of the circumference of the base and the height of the cylinder.

L.A. $= 2\pi rh$, or L.A. $= \pi dh$

The surface area of a right cylinder is the sum of the lateral area and areas of the two bases.

S.A. $=$ L.A. $+ 2B$, or S.A. $= 2\pi rh + 2\pi r^2$ (p. 702)

Theorem 11-3

Lateral and Surface Areas of a Pyramid

The lateral area of a regular pyramid is half the product of the perimeter p of the base and the slant height ℓ of the pyramid.

L.A. $= \frac{1}{2}p\ell$

The surface area of a regular pyramid is the sum of the lateral area and the area B of the base.

S.A. $=$ L.A. $+ B$ (p. 709)

Theorem 11-4

Lateral and Surface Areas of a Cone

The lateral area of a right cone is half the product of the circumference of the base and the slant height of the cone.

L.A. $= \frac{1}{2} \cdot 2\pi r\ell$, or L.A. $= \pi r\ell$

The surface area of a right cone is the sum of the lateral area and the area of the base.

S.A. $=$ L.A. $+ B$ (p. 711)

Theorem 11-5

Cavalieri's Principle

If two space figures have the same height and the same cross-sectional area at every level, then they have the same volume. (p. 718)

Theorem 11-6

Volume of a Prism

The volume of a prism is the product of the area of the base and the height of the prism.

$V = Bh$ (p. 718)

Theorem 11-7

Volume of a Cylinder

The volume of a cylinder is the product of the area of the base and the height of the cylinder.

$V = Bh$, or $V = \pi r^2 h$ (p. 719)

Theorem 11-8

Volume of a Pyramid

The volume of a pyramid is one third the product of the area of the base and the height of the pyramid.

$V = \frac{1}{3}Bh$ (p. 726)

Theorem 11-9

Volume of a Cone

The volume of a cone is one third the product of the area of the base and the height of the cone.

$V = \frac{1}{3}Bh$, or $V = \frac{1}{3}\pi r^2 h$ (p. 728)

Theorem 11-10

Surface Area of a Sphere

The surface area of a sphere is four times the product of π and the square of the radius of the sphere.

S.A. $= 4\pi r^2$ (p. 734)

Theorem 11-11

Volume of a Sphere

The volume of a sphere is four thirds the product of π and the cube of the radius of the sphere.

$V = \frac{4}{3}\pi r^3$ (p. 735)

Theorem 11-12

Areas and Volumes of Similar Solids

If the scale factor of two similar solids is $a : b$, then

- the ratio of their corresponding areas is $a^2 : b^2$, and
- the ratio of their volumes is $a^3 : b^3$. (p. 743)

Chapter 12 Circles

Theorem 12-1

If a line is tangent to a circle, then the line is perpendicular to the radius at the point of tangency. (p. 762)

Proof on p. 763

Theorem 12-2

If a line in the plane of a circle is perpendicular to a radius at its endpoint on the circle, then the line is tangent to the circle. (p. 764)

Proof on p. 769, Exercise 30

Theorem 12-3

If two segments are tangent to a circle from a point outside the circle, then the two segments are congruent. (p. 766)

Proof on p. 768, Exercise 23

Theorem 12-4

Within a circle or in congruent circles, congruent central angles have congruent arcs. (p. 771)

Proof on p. 777, Exercise 19

Converse

Within a circle or in congruent circles, congruent arcs have congruent central angles. (p. 771)

Proof on p. 778, Exercise 35

Theorem 12-5

Within a circle or in congruent circles, congruent central angles have congruent chords. (p. 772)

Proof on p. 777, Exercise 20

Converse

Within a circle or in congruent circles, congruent chords have congruent central angles. (p. 772)

Proof on p. 778, Exercise 36

Theorem 12-6

Within a circle or in congruent circles, congruent chords have congruent arcs. (p. 772)

Proof on p. 777, Exercise 21

Converse

Within a circle or in congruent circles, congruent arcs have congruent chords. (p. 772)

Proof on p. 778, Exercise 37

Theorem 12-7

Within a circle or in congruent circles, chords equidistant from the center (or centers) are congruent. (p. 772)

Proof on p. 773

Converse

Within a circle or in congruent circles, congruent chords are equidistant from the center (or centers). (p. 772)

Proof on p. 778, Exercise 38

Theorem 12-8

In a circle, if a diameter is perpendicular to a chord, it bisects the chord and its arc. (p. 774)

Proof on p. 777, Exercise 22

Theorem 12-9

In a circle, if a diameter bisects a chord (that is not a diameter), it is perpendicular to the chord. (p. 774)

Proof on p. 774

Theorem 12-10

In a circle, the perpendicular bisector of a chord contains the center of the circle. (p. 774)

Proof on p. 778, Exercise 33

Theorem 12-11

Inscribed Angle Theorem

The measure of an inscribed angle is half the measure of its intercepted arc. (p. 780)

Proofs on p. 781; p. 785, Exercises 26, 27

Corollary 1

Two inscribed angles that intercept the same arc are congruent. (p. 782)

Proof on p. 786, Exercise 31

Corollary 2

An angle inscribed in a semicircle is a right angle. (p. 782)

Proof on p. 786, Exercise 32

Corollary 3
The opposite angles of a quadrilateral inscribed in a circle are supplementary. (p. 782)
- Proof on p. 786, Exercise 33

Theorem 12-12
The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc. (p. 783)
- Proof on p. 786, Exercise 34

Theorem 12-13
The measure of an angle formed by two lines that intersect inside a circle is half the sum of the measures of the intercepted arcs. (p. 790)
- Proof on p. 791

Theorem 12-14
The measure of an angle formed by two lines that intersect outside a circle is half the difference of the measures of the intercepted arcs. (p. 790)
- Proofs on p. 796, Exercises 35, 36

Theorem 12-15
For a given point and circle, the product of the lengths of the two segments from the point to the circle is constant along any line through the point and circle. (p. 793)
- Proofs on p. 793; p. 796, Exercises 37, 38

Theorem 12-16
An equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$. (p. 798)
- Proof on p. 799

Constructions

Construction 1
Congruent Segments
Construct a segment congruent to a given segment. (p. 43)

Construction 2
Congruent Angles
Construct an angle congruent to a given angle. (p. 44)

Construction 3
Perpendicular Bisector
Construct the perpendicular bisector of a segment. (p. 45)

Construction 4
Angle Bisector
Construct the bisector of an angle. (p. 45)

Construction 5
Parallel Through a Point Not on a Line
Construct the line parallel to a given line and through a given point that is not on the line. (p. 182)

Construction 6
Quadrilateral With Parallel Sides
Construct a quadrilateral with one pair of parallel sides of lengths a and b. (p. 183)

Construction 7
Perpendicular Through a Point on a Line
Construct the perpendicular to a given line at a given point on the line. (p. 184)

Construction 8
Perpendicular Through a Point Not on a Line
Construct the perpendicular to a given line through a given point not on the line. (p. 185)